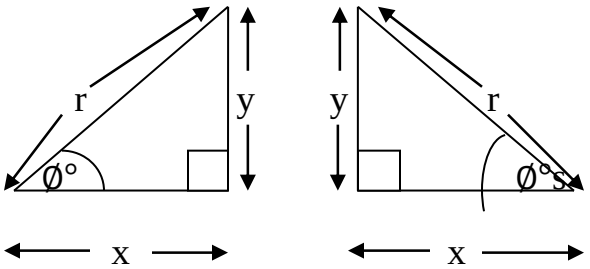
CHAPTER TEN

TRIGONOMETRY

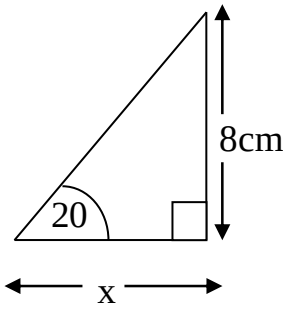


Consider the above two figures. For any one of them, the following facts must be noted:

1. When the length y is divided by the length x, we always get the tangent or the tan of the angle $\emptyset$, ie for any of the above figures $\tan\emptyset = \frac{y}{x}$.
2. When the length r is multiplied by the cosine or the cos of the angle $\emptyset$, we always get the length $x, ie\ r\ x\ cos\emptyset = x, \Longrightarrow \cos\emptyset = \frac{x}{r}$
3. When the length r is multiplied by the sine or sin of the angle $\emptyset$, we always get the length $y\ ie\ r\ x\ sin\emptyset\ =\ y, \Longrightarrow Sin\emptyset = \frac{y}{r}$

The use of tangent:

Q1.



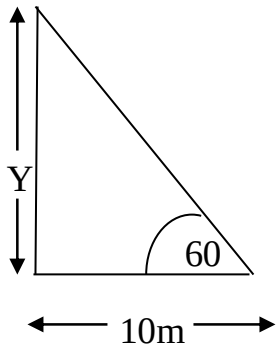
Find the value of x.

Soln.

Y = 8cm and $\emptyset = 20°, \Longrightarrow tan\emptyset = \frac{y}{x} \Rightarrow tan20° = \frac{8}{x} \Rightarrow x \times tan20° = 8, \Longrightarrow x\ tan20° = 8.$

Dividing through using tan 20° $=> \frac{x \tan 20^0}{\tan 20°} = \frac{8}{\tan 20°} \Longrightarrow x = \frac{8}{\tan 20^0} = \frac{8}{0.364}$, (since tan 20°) $= 0.364 \therefore x = 219cm.$

Q2.



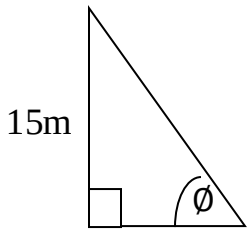
Find the length y.

Soln.

$\emptyset = 60°\ and\ x = 10m.$ Since $\tan \emptyset = \frac{y}{x} \Longrightarrow \tan 60° = \frac{y}{10}, \Longrightarrow 10 \times \tan 60° = y, \Longrightarrow y = 10 \times \tan 60°, => y = 10 \times 1.730, (since \tan 60° = 1.730) \Rightarrow y = 10 \times 1.730 = 17.3.$

∴Y = 17.30m.

Q3.



Calculate the angle Ø°

Soln.

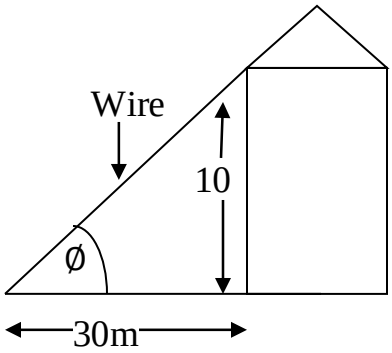
$y = 15m\ and\ x = 35m. From\ tan\emptyset = \frac{y}{x} \Longrightarrow \tan \emptyset = \frac{15}{35},$

$\Rightarrow \tan \emptyset = 0.428, \Rightarrow \emptyset = tan^{-1}\, 0.428,$

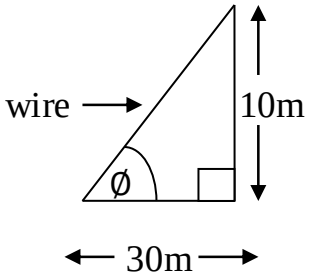
$=> \emptyset = 23°$ approx.

Q4. One end of a wire is fixed to a point 10m up a building. The other end is fixed to the ground at a point 30m away from the building. Find the angle which the wire makes with the ground.

Soln.



The above diagram can be represented as shown next:.

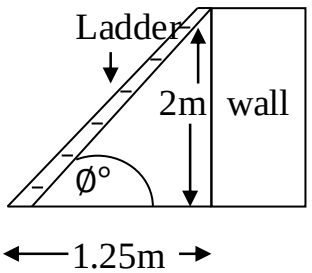


Let $\emptyset =$ the angle made by the wire with the ground.

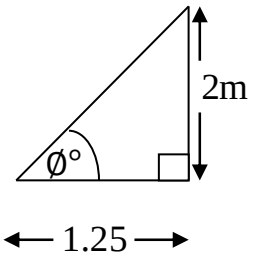
$tan\emptyset° = \frac{10}{30} \Longrightarrow tan\ \emptyset = 0.30 \Longrightarrow \emptyset = tan^{-1}0.30, \Longrightarrow \emptyset = 17° \therefore$
the wire makes an angle of $17°$, with the ground.

Q5. A ladder leans against a wall. The foot of the ladder is on the same horizontal level as the foot of the wall and is 1.25m away from it. The top of the ladder just reaches the top of the wall which is 2m high. Calculate the angle between the ladder and the ground.

Soln



The above figure can be simplified as shown next:



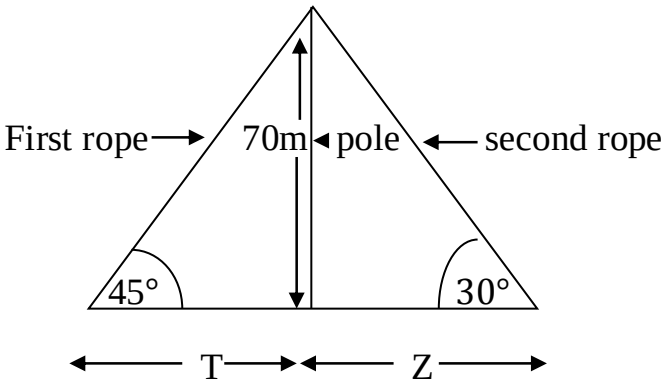
Let $\emptyset$ = the angle between the ground and the ladder.

Then tan $\emptyset° = \frac{2}{1.25} => tan\ \emptyset° = 1.6, \Longrightarrow \emptyset = tan^{-1}1.6 \Rightarrow \emptyset = 58°$The angle between the ladder and the ground = 58°

Q6. A pole is 70m long and stands on a leveled ground. One end of a first rope is tired to the top of this pole while the other end is fixed to a point on the ground, so that it makes an angle of 45° with the ground. One end of a second rope is fixed to the top of the pole, and its other end is fixed to the ground so that it makes an angle

of 30° with the ground. Calculate the distance between the points where the two ropes are fixed to the ground, if they are opposite to each other.

Soln



Let T = the distance between the foot of the pole and the point where the first rope makes an angle of 45° with the ground. Also let Z = the distance from the foot of the pole to the point where the second rope is fixed to the ground and makes an angle of 30° with the ground as shown in the diagram. Then the total distance between the two points = T + Z. We must therefore find T and Z

The original figure can be broken into two parts as shown next:

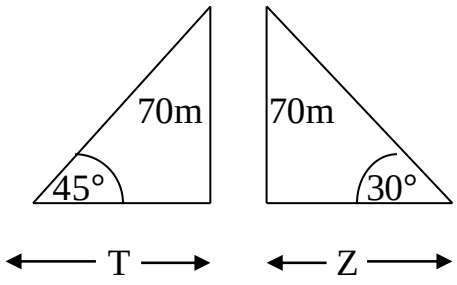


Figure (1) Figure (2)

From fig. (1), $\tan 45° = \frac{70}{T} \Longrightarrow T \times \tan 45° = 70°$,

$\therefore T \tan 45° = 70°$

Divide through using tan 45°. i.e

$\frac{T \tan 45°}{\tan 45°} = \frac{70}{\tan 45°} \Rightarrow T = \frac{70}{\tan 45°}$, but $\tan 45° = 1$

$\Rightarrow T = \frac{70}{1} = 70m.$

From fig. (2), $\tan 30° = \frac{70}{Z} \Longrightarrow Z \times \tan 30° = 70,$

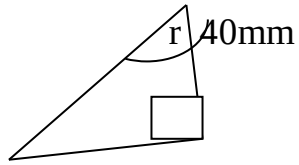
$\Rightarrow Z \tan 30° = 70$'s

Divide through using $\tan 30° \Longrightarrow \frac{Z \tan 30°}{\tan 30°} = \frac{70}{\tan 30°}$

$\Rightarrow Z = \frac{70}{\tan 30°}, but \tan 30° = 0.58, \therefore Z = \frac{70}{0.58} = 121,$

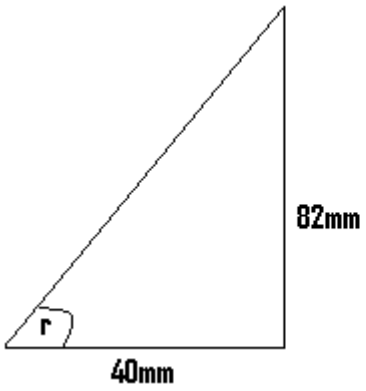
$\Rightarrow Z = 121m$. Distance between the two points = T + Z = 70 + 121 = 191m.

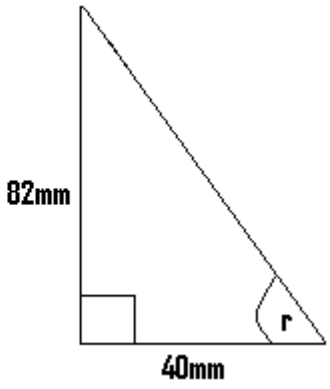
Q7. Calculate the angle marked r° in the figure below.



82mm

Rotate the figure to get any of the figures below (1.e rotate so that the location of angle rests horizontally).





By using any of the above figures, tan r° $= \frac{82}{40} \Longrightarrow tanr° = 2.05, => r° = tan^{-1}205 \Rightarrow r = 64.°$